Semiannual Progress Report No. 4

A STUDY OF SELECTED RADIATION AND PROPAGATION PROBLEMS RELATED TO ANTENNAS AND PROBE IN MAGNETO-IONIC MEDIA

Grant No. NsG 395

The National Aeronautics and Space Administration

Period Covered

1 October 1964 to 31 March 1965

Antenna Laboratory
Department of Electrical Engineering
Engineering Experiment Station
University of Illinois
Urbana, Illinois

INTRODUCTION

This progress report covers the activities in the period starting October 1, 1964 to March 31, 1965. Most research subjects reported earlier are continued. In addition, a few new boundary value problems in plasma medium (see 7, 8, 9 and 10 below) are considered during this period as an expansion of the research work in this area. Currently two more graduate students joined this group to carry out this expanded work.

SUMMARY OF THE RESEARCH

1. Impedance of a Loop in a Magnetoplasma - G. L. Duff

The quasi-static impedance of a small loop in a magnetoplasma has been derived in the case of the steady magnetic field normal to the plane of the loop. This quasi-static impedance is identical with the free space impedance of a small loop. This fact demonstrates that the loop impedance is less dependent on the plasma parameter than the impedance of a short dipole. Furthermore, the first-order correction term of the loop impedance was found to be independant of the impressed steady magnetic field. This suggested the calculation of the impedance of a small loop in a uniaxial medium. This is an easier problem to solve than the quasi-static case, because the fields are expressable in closed form. It was found that the loop impedance for the uniaxial case approaches the free space impedance in the limit as the free space wave number approaches zero.

A calculation of the Poynting vector and complex power flow is underway for the uniaxial case in the far fields and will be compared with the input impedance of the loop.

A general z-directed filamentary current source is a uniaxial medium has been studied and it was found that the fields of such a source can be written by inspection if the free space fields are known.

It is proposed in the coming period to complete a study of the far field Poynting vector and to compare the results of the quasi-static approximation with those of a source in a uniaxial medium.

2. Construction of a Gas Discharge Tube for Antenna Measurements in a Magneto-Aniaotropic Plasma - Larry S. Nixon, G. L. Duff

After considering several different basic tube structures, it was finally decided to use a brush cathode tube as described by Persson in the NBS Report No. 8452. The primary reasons for this choice were:

1. The plasma obtained is very uniform.
2. The plasma electron temperature is very low ($1000^{o}K$).
3. The ion density will be in the range for the desired X, Y, and Z plasma parameters ($X \leq 10$, $f=10^{9}$).

A simple discharge tube employing one cheaply constructed brush cathode was built to determine if the desirable properties listed above would be realized in practice. The cathode for this tube was simply dime store straight pins dropped through copper mesh with the pin heads soft soldered to the mesh. The anode for this tube was a simple copper ring, and two Langmuir probes were included for plasma measurements.

This tube worked well initially, giving us the desired uniform, low temperature plasma with ion densities of the proper order of magnitude. However, because of the cheap construction sputtering became a problem, especially at lower pressures where the mean free path became greater, and at higher currents our cathode began to heat, causing concern for our soft solder construction.

Clearly the primary problem then was to construct a tube on these same physical principles but with good vacuum tube materials and techniques. One of the first decisions was to make the pins of tungsten. Tungsten rods of the proper length and diameter were ordered and an etching apparatus was constructed to point 100 pins simultaneously.

Initially it was thought that these tungsten pins could simply be silver soldered to a copper mesh in much the same fashion as the previous cathode construction. After some frustrating trials with this proposed technique, it became clear that this method left much to be desired. Construction of a jig to hold the pin points in a plane during the soldering process, preventing the silver solder from running down the pins, and cleaning the assemblage to facilitate reasonable pump-down times, all proved to be serious drawbacks to this method. Also, silver solder and copper mesh are not the best metals available for use in a vacuum tube cathode.

After some thought and consultation a much better construction process has been developed that shows every sign of success to this date. This method involves copper brazing all 1669 tungsten pins simultaneously to a molybdenum base plate. A jig to hold the pins in place was machined from compressed carbon. A thin layer of nickel is electroplated on the molybdenum base plate and the blunt tips of the pins. A copper paste is placed on the plate and the pins are held in place by the carbon jig as the whole assembly is heated to about $1100^{o}C$ in a hydrogen furnace.

Two cathodes required for our final tube design have already been successfully fabricated in this manner. The resulting cathodes should prove to be much better than the previous cathode constructed with dime store straight pins. Not only are the materials now being used much superior for cathode use, the pin points are now closer spaced (.060" centers) and sharper, thus, enhancing the field emission effects. The finished tube as pictured in Figure 1 should be in operation by June 1965. As is mentioned in Persson's report, this tube will have an anode of brush construction also to promote better anode contact with the plasma.

During the next few months work will be completed on construction of the tube. The coils needed to produce the desired magnetic fields will be designed and constructed. The dimensions of the loop antenna and the necessary impedance measuring apparatus will be decided upon and assembled. Measurements of the plasma parameters will be made and tabulated for various pressures, currents, gases, and so forth so that the plasma will be a known quantity for the impedance measurements.

3. Propagation of Electromagnetic Waves in Linear Passive Media - O. B. Kesler

Properties of linear passive media are investigated by using a phenomenological approach. Properties of fields that can propagate in a passive medium are postulated and from this properties of the constitutive relationship are deduced. A necessary positive real condition on the constitutive relationship is found and some of its simplications are considered. Also, the casuality condition which is necessary for realizable media is considered.

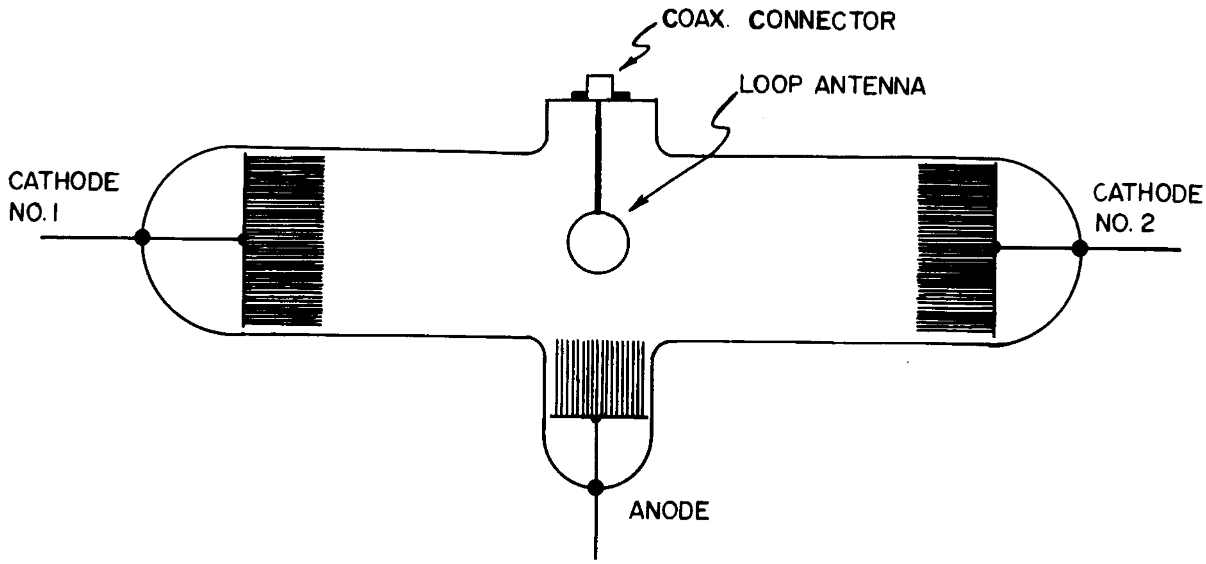

Figure 1.

Next a general formulation of the spectrum of characteristic waves in lossless linear passive media is made. Because of an orthogonality condition for the characteristic waves of the medium, the fields due to an arbitrary source can be separated into components parallel to the characteristic waves. The components of the source field are dependent only upon the portion of the of the source parallel to their characteristic field and to their own sheet(s) of the dispersion surface. The theory is then applied to two particular problems, electric dipoles in a general time-dispersive uniaxial medium and in an isotropic compressible plasma. Finally, the radiation field of an arbitrary source in a lossless linear passive medium is investigated using the spectral decomposition of the fields. By normalizing the length of the total Poynting vector (electromagnetic plus medium) to unity for each characteristic field, a concise and physically interpretable expression for the source fields is obtained. These results are then applied to an anisotropic compressible plasma and to a magneto-ionic plasma.

A technical report on this investigation has been prepared and will be issued shortly.

4. Biconical Antenna in an Anisotropic Medium - R. Mittra

In order to investigate several questions which have come up while studying the impedance properties of antennas in magneto-ionic media, it was felt that a study of the biconical geometry will be fruitful. The static problem in this geometry has recently been investigated by several authors, but no solution of the dynamic problem has been reported.

Two alternative approaches for formulating the problem are currently being investigated. The first one consists of the derivation of the wave equation in anisotropic medium corresponding to a spherical geometry. It has been possible to derive a coupled set of equations for the E_r and E_θ components which, however, are not separable.

An alternative attack in terms of an integral equation is also being investigated.

5. <u>Current Distribution and Input Impedance of a Thin Cylindric Antenna in Plasma</u> - Y. T. Lo

The complete mathematical solution in integral form for an infinite long cylindric antenna in anisotropic plasma was obtained earlier. As discussed in the previous report, when the radius of the antenna approaches zero, there is no residue contribution. During this period, a major effort is being made in the numerical evaluation of the poles for a nonvanishing radius which are given by the roots of a transcendental equation involving Hankel functions. This turns out to be a very difficult task. A program for these functions with complex argument has been written and tested. Then a second program was written for the transcendental equation in cooperation with the generalized Newton-Ralphson method for determining the roots. First, some difficulties are encountered in the convergence in iteration. Then another program based on steepest descent method was written; but the convergence problem remained unresolved. Lately, it was found that this difficulty is actually caused by the roundup error. Up to the present only one pair of complex roots has been accurately determined; unfortunately, they are the roots which can be determined analytically as reported earlier (namely under the case where $\alpha = \beta$), and also do not contribute to the integral. The work on searching other complex roots is being continued.

By Watson's Lemma, it can be shown that the current at a large distance from the generator varies as $\exp - j\, \gamma_\alpha\, |z|$, and $\exp - j\, \gamma_\beta\, |z|$, where $\gamma_{\alpha,\beta}$ are the branch points of the integrand and equal to $k_o[K' \mp K'')/\epsilon_o]^{1/2}$, k_o the free space wave number, ϵ_o the free space dielectric constant, K' and K'' the first diagonal and off-diagonal elements of the dielectric tensor, respectively, as indicated in the earlier report.

Under the condition that the dc magnetic field approaches infinity, the dielectric tensor becomes uniaxial and the results can be much simplified. This is currently under study.

6. <u>Study of Anisotropic Waveguides</u> - I. Akkaya, Y. T. Lo

The study of waveguides filled with anisotropic plasma as discussed in the previous report is being continued. The major problem at the moment is to determine numerically the roots of a transcendental characteristic equation for various values of plasma-frequency, and gyro-frequency, and radius of the circular guide. As discussed in the last report, cold plasma model ceases to be a valid approximation when the wave number becomes very large, e.g., near gyroresonance. Thus, in this case, the temperature effect of the plasma must be considered.

In the case of a warm plasma model, it is found that the modes that can exist in the waveguide can be determined by solving the following transcendental equation:

$$\begin{vmatrix} p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \\ p_3 & q_3 & r_3 \end{vmatrix} = 0$$

Where

$$p_i = -j \left[C_2^i \, (\nabla \pi_i)_r + C_1^i \, (\nabla \pi_i)_\phi \right]_{r=r_o}$$

$$q_i = \left[C_3^i \, k_t^i \; \pi_i \right]_{r=r_o}$$

$$r_i = \Big\{ \left[\gamma^2 + k_o^{\,2} - (k_t^i)^2 \right] C_2^i \, (\nabla \pi_i)_\phi + \left[-(\gamma^2 + k_o^{\,2}) \, C_1^i + j \, \gamma \, k_t^i \, C_3^i \, (\nabla \pi_i)_r \right] \Big\}_{r=r_o}$$

π_i's are potential functions,

r_o = the radius of the waveguide,

k_o = $2\pi/\lambda$, λ being the free space wave length,

$j\,\gamma$ = the propagation constant along the axis of the waveguide,

k_t^i = $k_t^i(\gamma)$, a function of γ, the form which depends upon the model chosen for the warm plasma,

and

C_j^i , (j = 1, 2, 3) , are the functions of γ and k_t, the forms of which also depend on the warm plasma model.

In order to take into account the interaction between the electromagnetic wave and the charged particles, a model based upon the Boltzmann equation and the Boltzmann distribution function is considered and the expressions for k_t^i 's and C_j^i 's are found. However, this particular model yields a dielectric tensor, the elements of which are given in terms of integrals which can be evaluated for the limiting case of low temperature and high dc magnetic field; but the expressions obtained in this way are again invalid for gyroresonance.

On the other hand, a plasma-filled circular cylindrical waveguide with a uniform dc magnetic field, which makes some angle α with the axis of the guide, was also considered. It is found that in this case in the guide there cannot exist modes independent of one another. But the solution of this problem can be given as the solution of an infinite set of linear equations of infinite unknowns, the coefficient of each unknown being an infinite series of integral forms and functions, the values of which can be found only by approximate or graphical methods. However, for the limit of very high dc magnetic field strength, these functions can be expressed explicitly.

7. Finite and Infinite H-plane Bifurcation Waveguide with Anisotropic Plasma Medium - S. W. Lee, Y. T. Lo, and R. Mittra

The problem of H-plane bifurcation of Parallel-plate waveguide with isotropic medium has been solved by Marcuvitz (1951), Hurd and Gruenberg (1954) and Mittra (1964) via different methods. In this investigation, the same problem is considered except that the waveguide is filled with anisotropic plasma.

The geometry of the problem to be considered is a rectangular guide of width a fitted with a perfectly conducting, infinitely thin septum starting from the plane $z = 0$ and extending to the plane $z = \ell$. The medium in the guide is the homogeneous cold plasma with a static B_o field parallel to the edge of the septum, i.e., along the y-axis. Under this assumed model, the medium is characterized by a relative dielectric tensor.

$$\underset{\approx}{K} = \begin{bmatrix} \epsilon_1 & 0 & -i\epsilon_2 \\ 0 & \epsilon_3 & 0 \\ i\epsilon_2 & 0 & \epsilon_1 \end{bmatrix}$$

where
$$\epsilon_1 = 1 - (\omega_p/\omega)^2 \left[1-(\omega_c/\omega)^2\right]^{-1}$$
$$\epsilon_2 = (\omega_p/\omega)^2 \left[(\omega/\omega_c) - (\omega_c/\omega)\right]^{-1}$$
$$\epsilon_3 = 1 - (\omega_p/\omega)^2$$

where ω_p and ω_c are the electron plasma and cyclotron frequencies, respectively. Only a two-dimensional problem with no y-variation is considered here. Starting with the Maxwell equation, it is readily shown that the wave equation for E_y and H_y are uncoupled. Therefore, we can consider separately the solution with $H_y = 0$ (TE mode with respect to z) and $E_y = 0$ (TM mode). The solution for the TE mode is trivial since it is identical to that when the guide is filled an isotropic medium with a relative dielectric constant ϵ_3.

The wave equation for H_y of TM mode is given by

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right] H_y \ (x,z) = 0 \tag{1}$$

where $k^2 = k_o^2 \ \epsilon/\epsilon_1$, $k_o^2 = \omega^2 \mu_o \epsilon_o$, $\epsilon^2 = \epsilon_1^2 - \epsilon_2^2$. It is clear that TE mode can propagate only when k^2 is a positive real number. This condition, in turn, sets a limit for the frequency ω, namely,

$$-\omega_c/2 + (\omega_p^2 + \omega_c^2/4)^{1/2} < \omega < (\omega_p^2 + \omega_c^2)^{1/2} \text{, lower band} \tag{2}$$

$$\omega_c/2 + (\omega_p^2 + \omega_c^2/4)^{1/2} < \omega < \infty \text{, upper band} \tag{3}$$

We shall confine our discussion to these two frequency ranges only. The electric field of TE mode is given by

$$E_x(x,z) = \left[h_1 \frac{\partial}{\partial x} - h_2 \frac{\partial}{\partial z}\right] H_y(x,z)$$

$$E_z(x,z) = \left[h_1 \frac{\partial}{\partial z} + h_2 \frac{\partial}{\partial x}\right] H_y(x,z)$$

where $h_1 = -\epsilon_2/\omega\epsilon_o\epsilon$ and $h_2 = i\epsilon_1/\omega\epsilon_o\epsilon$. First, let us consider the case of infinite bifurcation, i.e., $\ell = \infty$, and assume for simplicity that only the dominant mode propagates in the large guide A, and the z-component of the incident electric field is

$$E_z^i = \sin\frac{\pi}{a} x \quad e^{-\gamma_{1a} z} \tag{4}$$

where $\gamma_{1a} = [(\pi/a)^2 - k^2]^{1/2}$ is a negative imaginary number. The boundary condition to be satisfied is (1) $E_z(x,z) = 0$ at $x = 0$, a for all z; (2) $E_z(x,z)$ is continuous at $x = b$ for all z; (3) the total tangential electric field $E_z^t(x,z) = E_z^i + E_z$ vanishes on the septum $x = b$, $0 < z < \infty$, and (4) $H_y(x,z)$ is continuous at $x = b$, $z < 0$. Moreover, the usual edge condition that the current density normal to the edge vanishes at the edge as $[(x-b)^2 + z^2]^{1/2}$ is assumed.

Under these conditions one can solve Equation (1) for H_y by the Wiener-Hopf technique. Through some tedious manipulations, the solutions are given below:

$$H_y(x,z) = \sum_{n=1}^{\infty} \sin\frac{\pi b}{a} \quad \frac{h_2}{h_1^2 + h_2^2} \frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})} \frac{1}{L_+(i\gamma_{na})} \frac{1}{H_-^{\dagger}(i\gamma_{na})} \frac{1}{(i\gamma_{na} + i\gamma_{1a})}$$

$$\frac{1}{\sin\frac{n\pi}{a} b}\left[\frac{n\pi}{a}\cos\frac{n\pi}{a} x - \frac{h_1\gamma_{na}}{h_2}\sin\frac{n\pi}{a} x\right] e^{\gamma_{na} z}, \quad \text{for } z < 0 \tag{5}$$

and

$$H_y(x,z) = \sin\frac{\pi b}{a}\frac{h_2}{h_1^2 + h_2^2}\frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})}\frac{1}{H_-(a_o)}\frac{1}{a_o + i\gamma_{1a}}\frac{\gamma_o}{\sinh\gamma_o C} e^{\gamma_o(x-a)} e^{-ia_o z}$$

$$+ \frac{h_2}{h_1^2 + h_2^2}\frac{1}{L(-i\gamma_{1a})}\left[\frac{\pi}{a}\cos\frac{\pi}{a}(x-a) + \frac{h_1\gamma_{1a}}{h_2}\sin\frac{\pi}{a}(x-a)\right] e^{-\gamma_{1a} z}$$

$$+ \left\{\sin\frac{\pi b}{a}\frac{h_2}{h_1^2 + h_2^2}\frac{H_-(-i\gamma_{1a})}{L_-(-i\gamma_{1a})}\right\}$$

$$\left\{\sum_{n=1}^{\infty}\frac{(-1)^n}{L_+(-i\gamma_{nc})H_+(-i\gamma_{nc})}\frac{1}{(\gamma_{nc} - \gamma_{1a})}\left[\frac{n\pi}{C}\cos\frac{n\pi}{C}(x-a) + \frac{h_1\gamma_{nc}}{h_2}\sin\frac{n\pi}{C}(x-a)\right] e^{-\gamma_{nc} z}\right\}$$

for $z > 0$, $x > b$

where

$$L_+(a) = L_-(a) = (a_o - a)\,, \qquad a_o = k_o\epsilon_1^{1/2}\,, \qquad \gamma_o = k_o\epsilon_2\epsilon_1^{1/2}$$

$$H_+(a) = H_-(-a) = e^{-\Gamma(a)}\prod_{n=1}^{\infty}\frac{(a/n\pi)(\gamma_{na} - ia)}{(b/n\pi)(\gamma_{nb} - ia)(c/n\pi)(\gamma_{nc} - ia)} \tag{6}$$

$$\Gamma(a) = (ia/\pi)(a\,\ell n\, a - b\,\ell n\, b - c\,\ell n\, c)$$

$$\gamma_{nd} = \left[(n\pi/d)^2 - k^2\right]^{1/2}, \quad \text{for } d = a,\, b,\, c$$

For $z > 0$ and $x < b$, $H_y(x,z)$ have the same expressions as given above except that $(x-a)\, n\pi/c$, γ_{nc}, and c are replaced by x, $n\pi/b$, γ_{nb}, and b, respectively, and the sign of all terms are changed. Other field components can be obtained accordingly. It may be noted that the modes with the variation of $\exp \gamma_o z$, $\exp - i\, \alpha_o z$ type are TEM because the corresponding $E_z(x,z)$ is identically zero. However, it is different from the ordinary TEM mode of an isotropic guide in the following ways: (a) rather than being independent of the transverse dimension, it varies exponentially along x; (b) in the scattered field given by Equation (6), it is not continuous at $x = b$, and (c) whether it propagates or not depends solely on the properties of the plasma medium. Explicitly, it is propagated if $\alpha_o = k_o \epsilon_1^{1/2}$, is a negative real number, or $\omega > (\omega_p^2 + \omega_c)^{1/2}$. Because of the relation $\omega_c/2 + (\omega_p + \omega_c^2/4)^{1/2} \geq (\omega_p^2 + \omega_c)^{1/2}$, we conclude that for the frequency in the upper band defined by (3) the TEM mode propagates. Conversely, for frequency in the lower band defined by Equation (2), the TEM mode attenuates.

Since the TEM mode has zero $E_z(x,z)$ component, it is seen that the incident field in the above analysis is not uniquely specified by merely giving E_z^i, as in Equation (4). However, this does not affect the uniqueness of our solution as far as the scattered field is concerned, because an incident TEM mode does not give rise to any scattered field since it propagates freely in the guide without being disturbed by the septum.

For the case of finite bifurcation, an approximate solution can be obtained for large ℓ. However, due to the complexity, it is not given here, and the readers may be referred to the technical report by Lee, Lo and Mittra (1964).

References

1. Marcuvitz, N., Waveguide Handbook, McGraw-Hill Company, Inc., New York, 1951.

2. Hurd, R. A., and Gruenberg, H., "H-plane Bifurcation of Rectangular Waveguides," Can. J. Physics, 32, November 1954

3. Mittra, R., "Relative Convergence of the Solution of a Doubly Infinite Set of Equations," J. of Research, NBS, Vol. 67D, No. 2, March-April 1963.

4. Mittra, R., "The Finite Range Wiener-Hopf Integration Equation and a Boundary Value Problem in Waveguide," IRE Trans., PGAP, Vol. AP-7, Special Supplement, pp. S244-S254, December 1959.

5. Lee, S. W., Lo, Y. T., and Mittra, R., "Finite and Infinite H-Plane Bifurcation of Waveguide with Anisotropic Plasma Medium," Antenna Laboratory Report 65-7, Grant NsG 395, University of Illinois, March 1965.

8. H-plane Bifurcation of Waveguide with Compressible Plasma Medium - S. W. Lee, Y. T. Lo

In this investigation, the classical problem of H-plane bifurcation is considered when the medium in the guide is a homogeneous, compressible plasma. The plasma is treated as an electron gas. The motion of ions and dissipative effects such as collisions are neglected. Also, it is assumed that signals and perturbation are sufficiently small for the linearized fluid equations to be valid.

Starting with the following basic equations: (1) equation of motion, (2) equation of continuity, (3) equation of state, and (4) Maxwell equations, one can obtain two uncoupled equations for magnetic field and pressure:

$$\left[\nabla^2 + k_e^2\right] \underset{\sim}{H} = 0$$

$$\left[\nabla^2 + k_p^2\right] P = 0$$

where

$$k_e^2 = k_o^2 \epsilon_1 \,,\quad k_o^2 = \omega^2/\mu_o\epsilon_o \,,\quad \epsilon_1 = 1 - (\omega_p/\omega)^2 \quad \text{and} \quad k_p^2 = \omega^2\epsilon_1/V_s^2$$

The other two field quantities $\underset{\sim}{E}$ and $\underset{\sim}{V}$ are given by

$$\underset{\sim}{E} = \frac{1}{\epsilon_o\epsilon_1\omega} (\nabla \times \underset{\sim}{H}) - \frac{e}{m\,\epsilon_o\epsilon_1\,\omega^2} \nabla P$$

$$\underset{\sim}{V} = \frac{e}{m\,\epsilon_o\epsilon_1\,\omega^2} (\nabla \times H) - \frac{1}{i\,N_o\,m\,\epsilon_1\,\omega} \nabla P$$

First, assume that the incident wave is one of the propagating modes of the large guide of width 2a, namely,

$$H_y^i(x,z) = \left[\sinh \gamma_{eo}x + f_1(\alpha_o) \cosh \gamma_{eo}x\right] e^{i\alpha_o z}$$

where

$$\gamma_{eo} = (\alpha_o^{\,2} - k_o^{\,2})^{1/2}$$

$$\gamma_{po} = (\alpha_o^{\,2} - k_p^{\,2})^{1/2}$$

$$\triangle_1(\alpha_o)\ \triangle_2(\alpha_o) = 0$$

$$\triangle_1(\alpha) = [h_1\ h_4\ \gamma_e\ \gamma_p \sinh \gamma_p a \cos \gamma_e a + h_2\ h_3\ \alpha^3 \cosh \gamma_p a \sinh \gamma_e a]$$

$$\triangle_2(\alpha) = [h_2\ h_3\ \alpha^2 \sinh \gamma_p a \cosh \gamma_e a + h_1\ h_4\ \gamma_e\ \gamma_p \sinh \gamma_e a \cosh \gamma_p a]$$

h_1, h_2, h_3 and h_4 are parameters of the plasma medium.

By Wiener-Hopf technique, the solution for scattered field has been obtained but is too complicated to be included here. For numerical example, the characteristic roots of $\triangle_1(\alpha)$ and $\triangle_2(\alpha)$ must be solved first, and this will be performed at the next stage of investigation.

9. Waveguides Filled With Warm Plasma - C. Liang, Y. T. Lo

The objective of this study is to analyze the field behaviors inside a waveguide either of rectangular or of circular cross section which is filled with warm (compressible) plasma. Ultimately our interest will be centered upon the coupling between electromagnetic and acoustic waves, the fields produced by an arbitrary source in the guide, and the impedance prescribed at the input of the source. Since a plasma-filled guide with perfect conducting walls is physically realizable, it is hoped that experimental investigations can subsequently be performed to verify the analytical results, obtained under the assumed boundary conditions. It is generally assumed that the electron fluid cannot penetrate the guide wall. Without this assumption, the flux transport across the metal boundary will no doubt create a charge separation in the plasma and, thus a plasma sheath at the boundary. Nevertheless, we will make use of the simple but rather dubious conditions at the metal boundary, but it is of interest to note any deviation between our analytical results and the actual experimental observations.

Currently, we are investigating the source-free field solution inside a plasma-filled guide assuming that the tangential electric fields vanish on the walls and that there is no electron flux transport across the metal boundary. The plasma medium is described by the one-component (electron) fluid model with the motion of the heavier ions neglected, with no external magnetic field applied, and without collision between particles. Of course, in the fluid model the pressure is assumed to be a scalar, proportional to the square of the average thermal velocity times the number of electrons in the system. With the usual linearization process, we get the following basic equations (with $e^{-i\omega t}$ time variation):

$$\nabla \times \underset{\sim}{E} = i\omega\mu_o \underset{\sim}{H}$$

$$\nabla \times \underset{\sim}{H} = i\omega \epsilon_o \underset{\sim}{E} - n_o e \underset{\sim}{u}$$

$$i \omega P = n_o \omega V_s^2 (\nabla \cdot \underset{\sim}{u})$$

$$\underset{\sim}{u} = - i \left[\frac{e}{m\omega} \underset{\sim}{E} + \frac{1}{n_o m \omega} \nabla P \right]$$

Further, if we let $\underset{\sim}{H} = \underset{\sim}{H}_o$; $\underset{\sim}{E} = \underset{\sim}{E}_o + \underset{\sim}{E}_p$, we can easily separate the perturbations into two modes with exp γz variation along the axis of the guide

(1) <u>EM mode</u>:

$$(\nabla^2 + k_o^2) \underset{\sim}{E}_o = 0 , \qquad \nabla \cdot \underset{\sim}{E}_o = 0$$

$$(\nabla^2 + k_o^2) \underset{\sim}{H}_o = 0 , \qquad \nabla \cdot \underset{\sim}{H}_o = 0$$

where $$k_o^2 = \gamma^2 + \omega^2 \mu_o \epsilon_o (1 - \omega_p^2/\omega^2)$$

(2) <u>P mode</u>:

$$(\nabla^2 + k_p^2) P = 0 \qquad \text{where } k_p^2 = \gamma^2 + \frac{1}{\sigma_s^2} (\omega^2 - \omega_p^2)$$

$$\underset{\sim}{E}_p = \frac{\omega_p^2}{n_o e(\omega^2 - \omega_p 2)} \nabla P$$

The solutions of the differential equations can be readily obtained by applying the appropriate boundary conditions. For each model solution, the propagation constant of the wave must be determined from a determinental equation which is quite involved.

After we have studied the source-free field behaviors in the rectangular as well as the circular plasma-filled waveguides, we are planning to begin the investigation of the excitation problem by inserting a simple source into the guide (e.g., a small dipole). We hope that this will lead us into a better understanding of the waves in a warm plasma and provide us some information of the assumed boundary conditions if the experimental verification is feasible.

10. Excitation of a Plasma Slab and Other Related Problems - R. Mittra

The purpose of the work reported in this section is to study the radiation and excitation phenomenon from geometries which involve various plasma configurations, isotropic and anisotropic. To this end we have formulated the problem of a parallel plane waveguide exciting an isotropic dielectric slab which corresponds to the case of an isotropic plasma layer. A function theoretic technique has been used to seek the solution to the problem. The procedure is different from the Wiener-Hopf technique commonly applied to solve such problems. The advantage of the present method is that it may be extended to other problems which do not fall in the Wiener-Hopf category. Some of these, including the case of the anisotropic plasma, will be investigated in the future.

A paper entitled "An Alternative Approach to the Solution of a Class of Wiener-Hopf and Related Problems," has been accepted for presentation at the EM Theory Symposium in Delft, to be held September 6 - 11, 1965.

11. Publications under Grant No. NsG 395

1. Deschamps, G. A. and Kesler, O. B., "Radiation Field of an Arbitrary Antenna in a Magneto-plasma," IEEE Trans. on Antennas and Propagation, Vol. AP-12, 783-785, November 1964.

2. Mittra, R. and Duff, G. L., "A Systematic Study of the Radiation Patterns of a Dipole in a Magneto-plasma Based on a Classification of the Associated Dispersion Surfaces," to appear in Radio Science, Vol. 69D, No. 5, May 1965.

3. Mittra, R. and Duff, G. L., "A Classification of the Dispersion Surfaces in a Magnetic-ionic Medium and a Study of the Associated Radiation Patterns," Scientific Report No. 2, Antenna Laboratory, University of Illinois, Urbana, October 1964.

4. Mittra, R., "Solution of a Ferrite Boundary Value Problem and Resolution of Lewin's Paradox," Scientific Report No. 3, Antenna Laboratory, University of Illinois, Urbana, (to be issued shortly).

5. Lee, S. W., Lo, Y. T. and Mittra, R., "Finite and Infinite H-plane Bifurcation of Waveguides with Anisotropic Plasma Medium," Scientific Report No. 4, Antenna Laboratory, University of Illinois (to be issued shortly).

6. Kesler, O. B., "Electromagnetic Field in Linear Passive Media," (tentative title), Scientific Report No. 5, Antenna Laboratory, University of Illinois (to be issued shortly).

12. Travel

Professor Y. T. Lo attended the NASA - University Program Review Conference, March 1 - March 3, 1965.

Professor R. Mittra is planning to present two papers at the EM Theory Symposium to be held in Delft, Netherlands, September 6 - 11, 1965.

13. Personnel

Name	Position	Percent of full time charge to subject contract
Inci Akkaya	Research Assistant	50
Graham Duff	Research Assistant	50
Oren B. Kesler	Research Assistant	50
Y. T. Lo	Associate Professor	16 2/3
R. Mittra	Associate Professor	10